Two-proton radioactivity from excited states of proton-rich nuclei within Coulomb and Proximity Potential Model*

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We extend the Coulomb and Proximity Potential Model (CPPM) to study two-proton ($2p$) radioactivity from excited states while the proximity potential is chosen as AW95 proposed by Aage Withner in 1995. Demonstration reveals that the theoretical results acquired by CPPM exhibit a high level of consistency with prior theoretical models, such as the unified fission model (UFM), generalized liquid drop model (GLDM) and effective liquid drop model (ELDM). Furthermore, within the CPPM, we predicted the half-lives of potential $2p$ radioactive nuclei for which experimental data are currently unavailable. The predicted results were then assessed, compared with UFM, ELDM, and GLDM models, and examined in detail.

Keywords: $2p$ radioactivity, CPPM, half-lives, excited state

I. INTRODUCTION

Except for the well-known α, β, γ and cluster decays, etc[1–10], there exist some exotic modes of radioactivity in proton-rich nuclei [11–13], such as proton and two-proton ($2p$) radioactivity [14–18]. $2p$ radioactivity is an extremely exotic mode that is energetically possible in less massive nuclei near the proton-drip line [19–25], and it can reveal abundant information of nuclear structure, including the nuclear radius, wave function of the emitted two protons, spin and parity, deformation effect and so on [26–28]. This novel decay mode was firstly predicted by Zel'dovich and Goldansky in 1960s [29–31]. Based on the pioneering work of Zel'dovich and Goldansky, an extensive range of theoretical models have been proposed to describe this exotic decay process [32–35]. However, due to the limitations in radioactive beam facilities and detection technology, $2p$ radioactivity was not experimentally confirmed until the observation of ^{45}Fe $\rightarrow$ ^{43}Cr+p+p decay by Giovinazzo *et al.* at GANIL (France) and independently by Pfützner *et al.* at GSI (Germany) in 2002, which provided the first experimental evidence of this decay mode [36, 37]. Since then $2p$ radioactivity has been recognized as the significant decay mode for proton-rich nuclei [38], and it has been detected and studied in several nuclei, such as the resonant ground state of ^{6}Be [39, 40] and ^{12}O [41], and in the excited state selectively populated in ^{6}Be [40] and ^{14}O [42], etc.

Numerous studies have shown that the $2p$ radioactivity not only occurs from ground state but also from short-lived excited state. Jänecke was the first to discuss the possibility of β-delayed $2p$ ($\beta 2p$) radioactivity [38], while Goldansky predicted the occurrence of $\beta 2p$ radioactivity could be found in $\beta 2p$ emitters of $Z = 10 \sim 20$. In 1983, Cable *et al.* reported the first experimental observation of $\beta 2p$ radioactivity [43]. Subsequently, an increasing number of $\beta 2p$ emitters have been detected using silicon detector telescopes, making it possible to measure the energy of two individual protons with high precision. Since Cable *et al.* [43] discovered the $\beta 2p$ radioactivity from ^{22}Al, shortly they observed more $\beta 2p$ radioactivity from ^{26}P [44] and ^{35}Ca [45]. Hereafter, several other $\beta 2p$ nuclei were found, including ^{23}Si [46], ^{27}S [47], ^{31}Ar [48], ^{39}Ti [49], ^{43}Cr [50, 51] and ^{50}Ni [52]. In addition to populating excited state $2p$ radioactivity via β decays, $2p$ radioactivity has also been observed from excited state fed by nuclear reactions such as pick up, transfer or fragmentation, including ^{14}O [42], ^{17,18}Ne [53–57], ^{22}Mg [58, 59] and ^{28,29}S [60, 61]. In 2006, Mukha *et al.* first reported $2p$ radioactivity from ^{94}Ag in an experiment at GSI [62].

From a theoretical perspective, several methods have been proposed over the past few decades to study the mechanism of $2p$ radioactivity, including both microscopic and phenomenological models over the previous decades. In general, there are three distinct ways for proton-rich nuclei to emit two protons: (1) two-body sequential emission, (2) three-body simultaneous emission, and (3) diproton emission (also called ^{2}He cluster emission). The ^{2}He cluster emission is an extreme scenario with the two strongly correlated protons that can only survive for a brief time before splitting after passing through the Coulomb barrier. Recently, based on the CPPM, Yao *et al.* [64] and Ghodsi *et al.* [65], as well as Deng *et al.* [66] and Santhosh *et al.* [67], performed comparative

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studies of various proximity potential formalisms to study α decay, proton radioactivity and cluster radioactivity, respectively. Considering that the $2p$ radioactivity decay process shares the same theory as α decay and proton radioactivity, i.e., barrier penetration [68–73], we extended the CPPM presented in Ref. [74] to account for ground state $2p$ radioactivity. It was found that both the calculated and predicted results were highly consistent with experimental data and results from other theoretical models. In fact, except for theoretical models the empirical formulae are excellent tools for investigating $2p$ radioactivity involving a four-parameter empirical formula provided by Sreeja *et al.* [34] and a two-parameter empirical formula proposed by Liu *et al.* [63]. Consequently, a desirable question is whether $2p$ radioactivity from excited states can be regarded as ^{2}He cluster emission and described by the CPPM. To address this question, we systematically studied the half-lives of $2p$ radioactivity from excited states within the CPPM for nuclei with $8 < Z < 47$ in this work.

This paper is organized as follows: In Sec. II, the theoretical framework of $2p$ radioactivity half-life and the CPPM formalism are described briefly. In Sec. III, detailed calculations and discussion are presented. Finally, a summary is given in Sec. IV.

II. THEORETICAL FRAMEWORK

A. The $2p$ radioactivity half-life formalism

The half-life of $2p$ radioactivity is generally determined by

$$T_{1/2} = \frac{ln2}{\lambda} = \frac{ln2}{\nu P S_{2p}}, \tag{1}$$

where λ is the decay constant. ν is the assault frequency related to the harmonic oscillation frequency presented in the Nilsson potential [75], it can be written as

$$h\nu = \hbar\omega \simeq \frac{41}{A^{1/3}}. \tag{2}$$

Here h, $\hbar$, A and ω are the Planck constant, reduced Plank constant, mass number of parent nucleus and angular frequency, respectively. $S_{2p} = G^2[A/(A-2)]^{2n}\chi^2$ represents the preformation probability of the emitted two protons in parent nucleus obtained by the cluster overlap approximation[74]. Here $G^2 = \frac{(2n)!}{2^{2n}(n!)^2}$ [20, 86] with $n \approx (3Z)^{1/3} - 1$ is the average principal proton oscillator quantum number and χ^2 is set as 0.0143 acroding to Ref. [23]. The penetration probability P can be calculated by the WKB approximation and written as

$$P = \exp\left[-2\int_{r_{in}}^{r_{out}} K(r)\, dr\right], \tag{3}$$

where $K(r) = \sqrt{\frac{2\mu}{\hbar^2}|V(r) - Q_{2p}|}$, μ denotes the reduced mass [35]. r is the mass center distance between the daughter nucleus and emitted two protons. r_{in} and r_{out} are classical inner and outer turning points which can be obtained from the conditions $V(r_{in}) = V(r_{out}) = Q_{2p}$. The whole interaction potential $V(r)$ between the emitted two protons and daughter nucleus is composed of the Coulomb potential $V_C(r)$, nuclear potential $V_N(r)$ and centrifugal potential $V_\ell(r)$. It can be expressed as

$$V(r) = V_N(r) + V_C(r) + V_\ell(r). \tag{4}$$

In CPPM, the nuclear potential is replaced by proximity potential. A detailed description of the proximity potential will be provided in Sec. II B.

Assuming a homogeneous spherical charge distribution for the daughter nucleus, the Coulomb potential $V_C(r)$ is postulated to be the potential of a uniformly charged sphere with radius R. It is expressed as follows:

$$V_C(r) = \begin{cases} \frac{Z_1 Z_2 e^2}{2R}[3 - (\frac{r}{R})], & r < R, \\ \frac{Z_1 Z_2 e^2}{r}, & r > R, \end{cases} \tag{5}$$

where $R = R_1 + R_2$ is the separation radius with R_1 and R_2 being the radii of daughter nucleus and emitted two protons, respectively.

For $V_\ell(r)$, we choose the Langer modified form, because $\ell(\ell+1) \to (\ell + \frac{1}{2})^2$ is a necessary correction for one-dimensional problem [76]. It can be written as

$$V_\ell(r) = \frac{\hbar^2(\ell + \frac{1}{2})^2}{2\mu r^2}, \tag{6}$$

where ℓ is the obrital angular momentum taken away by the emitted two protons.

B. The proximity potential formalism

The phenomenological proximity potential was first proposed by Blocki *et al.* [77] in 1970s for heavy-ion reactions in 1970s for the first time. It provides a simple formula for the nucleus-nucleus interaction energy as a function of the separation between the surfaces of the approaching nuclei, with adjustable parameters that makes use of the measured values of the nuclear surface tension and surface diffuseness. In the CPPM, the potential energy barrier is modeled as the sum of Coulomb potential, proximity potential and centrifugal potential for both the touching configuration and separated fragments. In this work, we select the AW95 proposed by Aage Withner in 1995 [78] as an example to replace nuclear potential. This set of proximity potential can be expressed as

$$V_N(r) = -\frac{V_0}{1 + \exp\left(\frac{r - R_0}{0.63}\right)}. \tag{7}$$

Here $V_0 = 16\pi\frac{R_1 R_2}{R_1 + R_2}\gamma a$ with

$$a = \frac{1}{1.17\left(1 + 0.53\left(A_1^{-1/3} + A_2^{-1/3}\right)\right)} \quad \text{fm}. \tag{8}$$

$R_i = (1.2A_i^{1/3} - 0.09)$ fm $(i = 1, 2)$ $R_0 = R_1 + R_2$. The surface energy constant γ has the form as

$$\gamma = \gamma_0 \left[1 - k_s \left(\frac{N_1 - Z_1}{A_1} \right) \left(\frac{N_2 - Z_2}{A_2} \right) \right], \tag{9}$$

where the coefficients γ_0 = 0.95 Mev/fm^2 and K_s = 1.8. Here A_i, Z_i and N_i $(i = 1, 2)$ are the mass number, proton number and neutron number daughter nucleus and emitted two protons, respectively.

III. RESULTS AND DISCUSSION

Blocki first presented the proximity potential in 1977 to characterize the interaction potential between any two nuclei in the separation degree of freedom, based on the proximity force theorem [77]. Hence, numerous nuclear proximity potentials have been widely used in nuclear physics research [79–83]. The CPPM is a phenomenological model that is commonly used to study the two-body problem in the context of $2p$ radioactivity. It considers the interaction potential between the parent and daughter nuclei as the sum of the nuclear potential, Coulomb potential, and centrifugal potential. Unlike other models, the CPPM utilizes the proximity potential, which replaces the nuclear potential with a simplified formalism based on the proximity force. This provides the model with the advantage of adjustable parameters, making it simple yet accurate. In our previous work [74], we extended the CPPM to study $2p$ radioactivity from the ground state, the main intention of this work is to further extend the CPPM to investigate $2p$ radioactivity from excited states.

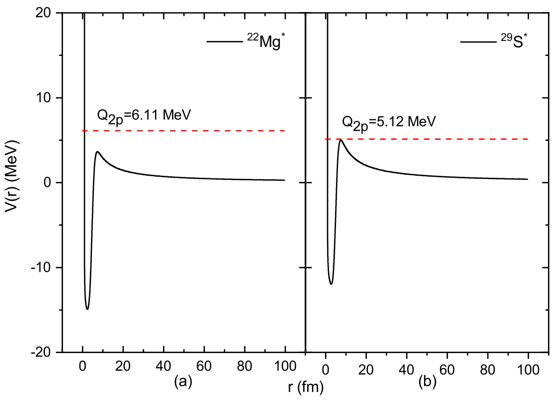

Fig. 1. (color online) The whole interaction potential of ^{22}Mg* and ^{29}S*

Firstly, we performed the calculations on $2p$ radioactivity half-lives for ^{14}O*, ^{17}Ne*, ^{18}Ne*, ^{22}Mg*, ^{29}S* and ^{94}Ag* (* represent the excited state), all of the calculated results are listed in the Table 1. In this table, the former four columns represent the $2p$ decay process, experimental 2p radioactivity released energy, spin and parity of the initial and final state of the nucleus, angular momentum taken away by the emitted two protons, respectively. The fifth column is the experimental half-lives of $2p$ radioactivity of excited state. From the sixth to ninth columns, they represent the logarithmic form of $2p$ half-lives obtained by CPPM, ELDM, GLDM and UFM, respectively. In general, from this table, it is obviously that the theoretical half-lives obtained by CPPM are highly consistent with other theoretical models, except for the nuclei ^{22}Mg* (with the $Q_{2p} = 6.11$ MeV) and ^{29}S* (with the $Q_{2p} = 5.12$ MeV). In order to intuitively explain this phenomenon, we plot the interaction potential curve of ^{22}Mg* and ^{29}S* in Fig. 1. In Sec. II A, the half-lives of $2p$ radioactivity are depended on penetration probability P which is obtained by Eq. 3 and conditions $V(r_{in}) = V(r_{out}) = Q_{2p}$.

Unfortunately, in Fig. 1, the whole interaction potential curve of ^{22}Mg* and ^{29}S* can't satisfied the conditions mentioned above, so that we can't calculate corresponding *P*, naturally, the half-lives can't be obtained.

For the reason of this phenomenon, we turn our attention to remarkable deformation effect. In fact, the half-lives of $2p$ radioactivity are highly sensitive to proton-proton interaction due to the paring effect of valence protons. The quasi-classical ^{2}He model can't account for the experimentally observed proton-proton correlations, which indicate back-to-back proton emission. Moreover, for the nucleus ^{22}Mg* and ^{29}S*, their spin-parity of the initial state of the parent nuclei are uncertain, and the the values of Q_{2p} are floating, all of these factors have nonnegligible impact on $V(r)$ curve.

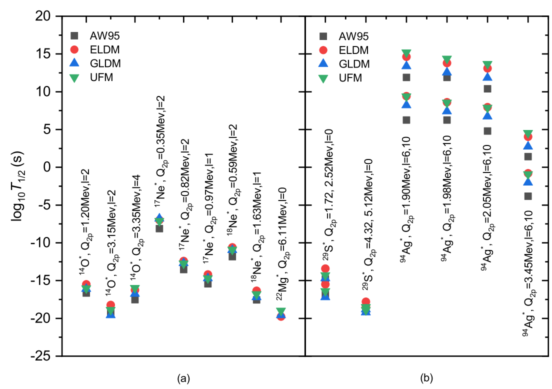

Fig. 2. (color online) Comparing the half-lives of the nuclei which obtained by CPPM in Table I with other theoretical models.

In order to describe the agreement of the half-lives for the $2p$ radioactivity of excited state which calculated by CPPM, partial theoretical results are plotted in Fig. 2. In this figure, the logarithmic half-lives of the $2p$ radioactivity of excited state which obtained by ELDM, GLDM and UFM are compared with our calculations [84, 85]. For the ^{14}O*, in Table 1, its experimental data are given as $\log_{10}T_{1/2} >$ -16.12 s, the calculated half-lives by CPPM are very close to those from ELDM, GLDM and UFM, simultaneously the calculated results are better than from the some empirical formulas such as R-matrix theory [42], Sreeja formula and Liu

formula [85], whose half-lives are -18.12 s, -19.94 s and -16.85 s, respectively. In order to intuitively discuss the influence of angular momentum ℓ and release energy Q_{2p} for the half-lives of the $2p$ radioactivity from excited state, we select ^{14}O* and ^{94}Ag* as an example to analyze their contribution to the half-lives. In Table 1, it is obviously find that the half-lives of ^{14}O* obtained by CPPM for the same value of ℓ nearly have three magnitudes deviation with different released energy Q_{2p} value (Q_{2p}= 1.20 MeV and Q_{2p}= 3.15 MeV). Simultaneously, for the ^{94}Ag*, identical Q_{2p} but different ℓ also have three magnitudes while the value of angular momentum are 6 to 10. In fact, this conclusion is same with our previous work [87]. It is obviously found that either Q_{2p} or ℓ has a nonnegligible contribution to the ^{14}O* and ^{94}Ag* for their corresponding theoretical half-lives within CPPM, ELDM, and GLDM. The reason for this phenomenon is the theory of these models are similar, the penetration probatbility P are both calculated by WKB approximation. In other word, these theoretical models have a strong dependency with Q_{2p} and ℓ, and we suspect the corresponding experimental data are not precise enough. Configuration mixing and three-body dynamics are commonly utilized in nuclear and molecular physics to describe systems with numerous interacting particles. However, due to the limitations of the phenomenological theoretical model, which fails to account for configuration mixing and three-body dynamics, our calculations treat the two emitted protons as strongly correlated, thus reducing them to a two-body problem. As a result, CPPM cannot provide comprehensive information on particle dynamics. Up to now, the experimental $2p$ radioactivity data of excited state are rare, the reason is that it's hard to observe this decay process due to the extremely short half-lives. From the scheme of these excited nuclei, it is evident that one-proton (1p) radioactivity also exists alongside two-proton radioactivity during their decay process, as well as three-proton (3p) radioactivity decay process. Therefore, the branching ratios between 1p and 2p decay serve as a crucial index for comprehending the abundant information on nuclear structure of these nuclei and merit further investigation.

IV. CONCLUSION

In this work, we extend the CPPM to study the excited $2p$ radioactivity of ^{14}O*, ^{17}Ne*, ^{18}Ne*, ^{22}Mg*, ^{29}S* and ^{94}Ag*. It is found that the theoretical values obtained by CPPM are highly consistent with corresponding experimental data and theoretical values obtained by ELDM and GLDM. For the nuclei ^{22}Mg* and ^{29}S* (with the $Q_{2p} = 5.12$ MeV), the CPPM is not suitable, the reason for this phenomenon perhaps caused by the angular momenta are not available because the spin-parity of the initial state of the parent nuclei has not determined. In addition, the uncertain values of Q_{2p} also provided strong influence on calculated results. Simultaneously, it is found that the half-lives of excited $2p$ radioactivity have a strong relationship with Q_{2p} and ℓ. Comparing with the theoretical results obtained by ELDM, GLDM and UFM, the half-lives of the excited $2p$ radioactivity by CPPM are reliable, it maybe as a positive guideline for future experiments.

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TABLE 1. Comparison between the experimental $2p$ radioactivity half-lives of excited state and those within different theoretical models.

$2p$ emission	Q_{2p} (MeV)	$j_i^\pi \to j_f^\pi$	ℓ	$\log_{10} T_{1/2}^{Expt.}$	$\log_{10} T_{1/2}^{CPPM}$	$\log_{10} T_{1/2}^{ELDM}$[84]	$\log_{10} T_{1/2}^{GLDM}$[84]	$\log_{10} T_{1/2}^{UFM}$[85]
$^{14}O^* \to {}^{12}C$	1.20	$2^+ \to 0^+$	2	>-16.12	-16.65	-15.49	-16.10	-16.02
$^{14}O^* \to {}^{12}C$	3.15	$2^+ \to 0^+$	2		-19.25	-18.22	-19.58	-18.87
$^{14}O^* \to {}^{12}C$	3.35	$4^+ \to 0^+$	4		-17.53	-16.25	-16.76	-15.96
$^{17}Ne^* \to {}^{15}O$	0.35	$3/2^- \to 1/2^-$	2	>-10.59	-8.11	-6.98	-6.79	-7.11
$^{17}Ne^* \to {}^{15}O$	0.82	$5/2^- \to 1/2^-$	2		-13.54	-12.41	-12.68	-12.73
$^{17}Ne^* \to {}^{15}O$	0.9 7	$3/2^- \to 1/2^-$	1		-15.41	-14.20	-14.68	-14.69
$^{18}Ne^* \to {}^{16}O$	0.59	$2^+ \to 0^+$	2		-11.82	-10.59	-10.96	-10.91
$^{18}Ne^* \to {}^{16}O$	1.63	$1^- \to 0^+$	1	$-16.15^{+0.06}_{-0.06}$	-17.56	-16.34	-17.20	-16.79
$^{22}Mg^* \to {}^{20}Ne$	6.11	$- \to 0^+$	0		-	-19.75	-19.58	-18.97
$^{29}S^* \to {}^{27}Si$	1.72-2.52	$- \to 5/2^+$	0		-16.851∼-14.78	-15.5∼-13.4	-14.7∼-17.2	-16.4∼-14.3
$^{29}S^* \to {}^{27}Si$	4.32-5.12	$- \to 5/2^+$	0		-∼-19.04	-18.4∼-17.8	-19.2∼-18.8	-18.9∼-18.5
$^{94}Ag^* \to {}^{92}Rh$	1.90	$21^+ \to 11^+$	6∼10	$1.90^{+0.38}_{-0.20}$	6.25∼11.89	9.42∼14.63	8.22∼13.38	9.38∼15.21
$^{94}Ag^* \to {}^{92}Rh$	1.98				5.45∼11.07	8.61∼13.80	7.41∼12.55	8.56∼14.37
$^{94}Ag^* \to {}^{92}Rh$	2.05				4.79∼10.39	7.95∼13.11	6.74∼11.86	7.89∼13.68
$^{94}Ag^* \to {}^{92}Rh$	3.45				-3.82∼1.41	-0.8∼4.04	-2.03∼2.75	-0.92∼4.56

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